coordinated\_ordering13

GroupB

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Table of Contents

# Introduction

Production planning is the process of determining a tentative plan for how much to produce in the next time period, during an interval of time called planning horizon. It is an important challenge for industrial companies because it has a strong impact on their performance in terms of customer service quality and operating costs. Production planning often proves to be a very complex task because of several reasons, some of which are discussed below. One of the reasons is variability of products. A production resource is not only used for the production of one product, but to produce different types of products. The production resources available are limited and inflexible, as they can be used to produce only one type of product at a given time with a given production rate. Though the efficiency of a production resource is aimed to be maximised, it can still be used for a single product at a given time. The production planner thus needs to decide which products should be produced, when and in what quantities, while taking into account all the constraints in the system. Another reason is that a production plan might have to meet several conflicting objectives, for example, minimizing production and inventory costs while simultaneously increasing the productivity and the flexibility of production facilities and guaranteeing excellent customer service levels. One major planning decision is finding the economic lot size (production or order) which balances the inventory, ordering and set up costs. Deciding which materials are required to realise an aggregate production plan per period as well as determining how to properly bundle production lots in order to save ordering costs and stock up cost constitutes some of the most important operational activities of industrial companies. Since the late 1940’s, inventory theory has been one of the most successful operational research techniques to be applied in business, industry and the public sector (Goyal and Gupta, 1989). The very first lot size model was proposed by Ford Whitman Harris’ in 1913 in his seminal paper entitled how many parts to make at once. In his paper, Harris proposed the Economic order Quantity model (EOQ) which enables the determination of optimum order quantities while avoiding excess inventory holding cost and shortage costs. Since its publications many variations have been made to this basic lot sizing problem including but not limited to: lot sizing models with dynamic or stationary demand, single or multiple level problems, single item or multiple item, capacitated or uncapacitated, deteriorating items, and deterministic or stochastic demand. Detailed description of these variations and more can be found in the papers by Karimi et al. (2003) and Christoph H. et al.(2014). This paper aims to determine an optimal approach for coordinating the orders for a multi item capacitated lot sizing problem with deterministic demand. The reasoning here is that although the EOQ is optimal for the buyer, the buyer may stand to benefit from ordering more than the EOQ in situations where this leads to decreased transportation costs, administrative and handling costs as well as increased price discounts from suppliers. In this paper, a case study of a single storage unit of flow rack with multiple levels is explored where the demand for various materials is deterministic. The rest of this paper is structured as follows: a detailed description of the problem will be given in chapter 2, and the separate ordering and joint ordering solution approaches will be presented in chapter 3. Chapter 4 will expatiate on the assignment of lanes based on the cutting stock problem and chapter 5 will show an analysis of the overall results and concluding remarks.

# Case Description

The given case describes an inbound warehouse where multiple materials are stored in a storage facility.We are concerned with the storage facility which is the continuous flow rack. A continuous flow rack is used to store small and medium sized parts. There are a total of 8 racks, each flow rack consists of 4 levels and multiple runways or lanes per level.

* *Materials and Boxes*

There are 62 materials in total and 9 types of boxes with varying dimensions. The materials are ordered and stored in boxes. There are different types of boxes, but each part is stored in one particular box type. Each box has a specific capacity for the number of items that can be stored in it.

In each runway only one part should be stored. Thereby, the width of a runway can be adapted to the width of the box type which corresponds to the part assigned to the runway. Thus, the total number of runways per level depends on the assigned parts and is limited by the total width of the rack.

* *Cost Distribution*

All parts are ordered from one supplier and only complete boxes can be ordered. The demand per day is given. Ordering of each box with a particular item incurs an ordering cost which is variable and dependent on the material ordered. A fixed ordering cost of €1500 is incurred every time at least one order is placed. The stock holding cost is based on the unit price and the interest rate.

* *Task*

The aim of the project is to determine a lot sizing model that determines lot sizes for each part such that rack capacity constraints are adhered to and the total cost per period for stock holding and ordering are minimal.

# Data preparation

Prior to analysing the data and formulating the models for the solution approaches, several manipulations were made to convert the data into a form that can be readily and accurately studied.

The following storage information is derived from the data sheets provided:

, , , ,

|  |  |  |  |
| --- | --- | --- | --- |
| Total\_racks | levels\_per\_rack | rack\_length | rack\_height |
| Rack Information | 8 | 4 | 6000 |

The following modifications were made to the data:

* *Product number (NO)*

Numbers were assigned to the materials ranging from 1 to 62 increasing order. The purpose of this is to make it easier to use and to refer to the products, rather than using their material IDs.

Storage Information

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Total\_racks | levels\_per\_rack | rack\_length | rack\_width | rack\_height |
| Rack Information | 8 | 4 | 6000 | 1750 | 300 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| N0 | material ID | demand per day | box ID | pieces/box | price |
| 1 | 7305667+74 | 12 | 6203060 | 45 | 1.51 |
| 2 | 7305669+77 | 30 | 6203059 | 15 | 7.62 |
| 3 | 7305670+77 | 30 | 6203059 | 15 | 1.62 |
| 4 | 7305673+76 | 30 | 6203059 | 16 | 1.51 |
| 5 | 7305674+76 | 30 | 6203059 | 16 | 1.51 |

## Convert to Boxes

From the data given, demand per item , quantity ordered per item , and the unit price per item , are all converted in terms of boxes, using the capacity of the box for part , that is,

* The demand per item is converted to demand per box by dividing demand per item by the capacity of the box for part .

\* The demand per box is transformed to yearly demand by multiplying it by 262 days. We assumed that there are 262 working days for the year 2020 according to the Iowa Working Day Payroll Calendar. demand per year

* The prices per item is converted into price per box , by multiplying the price of the items by the number of items in each box.

value of box

Merged data with yearly demand

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| box ID | N0 | material ID | box\_cost | ordering\_cost |
| 3103147 | 54 | 7348808+73 | 46.4 | 50 |
| 3103147 | 52 | 7346592+72 | 127.4 | 50 |
| 3104147 | 61 | 7455691+71 | 87.8 | 45 |
| 3104147 | 62 | 7455692+71 | 87.8 | 45 |
| 3106410 | 15 | 7305865+77 | 44.1 | 60 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| box ID | N0 | material ID | demand per day | box\_cost | ordering\_cost |
| 6203060 | 1 | 7305667+74 | 12 | 67.95 | 75 |
| 6203059 | 2 | 7305669+77 | 30 | 114.30 | 80 |
| 6203059 | 3 | 7305670+77 | 30 | 24.30 | 80 |
| 6203059 | 4 | 7305673+76 | 30 | 24.16 | 80 |
| 6203059 | 5 | 7305674+76 | 30 | 24.16 | 80 |

# Solution Approaches

* *Notation* The following notations were used while devising approaches. $i I $ denote the set of parts considered, Total number of parts

demand per day for part

demand per year for part , $d\_i 262 $ days

order quantity for part

unit price for part

box capacity

fixed ordering cost

variable ordering cost per box for part

stock holding cost rate based on unit price and interest rate therefore

*Assumptions*

*1.* Every demand is fulfilled *2.* Demand is deterministic *3.* Lead time is zero *4.* Waiting time is zero *5.* There is limited storage capacity *6.* Only complete boxes can be stored. In each runway, only one type of boxes are stored.

# Separate Ordering (SO)

In separate ordering, each box containing the unique material is ordered separately. Contrary to joint ordering, the orders are not batched together. Thus, each order incurs the fixed ordering cost of 1500 EUR in addition to the cost of holding and variable ordering cost.

The given case contains 62 different materials so in case of separate ordering, each of the materials are ordered separately. Thus 62 different order frequencies.

The data about materials and their expected demand per day has been provided in the excel sheet “Product data.xlsx”.

Firstly, to determine how much to order (order quantity) and when to place order (the reorder point), we turned to the EOQ model. EOQ Model can be used in this case because the problem description matches with the assumptions in an EOQ model. The information for the case with respect to assumptions in the EOQ model is listed below.

*1.* Demand is uniformly distributed accross 262 days. In this case, the expected demands are given.

*2.* Whenever an order is placed, a fixed cost is incurred. For the case, the fixed cost is denoted by which equals 1500 EUR.

*3.* Ordering cost for part is denoted by .

*4.* The stock holding cost is based on the unit price and interest rate , therefore it will cost $h p\_i $ to stock one unit of that item.

*5.*  The order arrives immediately after the placement of the order.

*6.*  All demand is satisfied on time.

## EOQ modelling

EOQ is calculated for each part i using the given formula which is denoted by Q\* or Q max (Q max notation is used to differentiate it from Q min notation used in the later part) and minimum EOQ is calculated by omitting the fixed ordering cost of 1500. This is done to find the starting feasible solution.

calculate EOQ for each part i:

$$ q\_i^{max} = \sqrt \frac{2 \cdot y\_i \cdot (c\_i^{or}+c^{-or})}{pr\_i \cdot h}$$

Min EOQ, used in starting feasible solution.

$$ q\_i^{min} = \sqrt \frac{2 \cdot y\_i \cdot (c\_i^{or})} {pr\_i \cdot h}$$

these notations can be used interchangably both meaning Maximum EOQ.

Adding both EOQs, “EOQ.max” and “EOQ.min” to the product\_data table:

Both EOQs added to the table

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| box ID | N0 | material ID | demand per day | ordering\_cost | eoq.min | eoq.max |
| 6203060 | 1 | 7305667+74 | 12 | 75 | 39 | 180 |
| 6203059 | 2 | 7305669+77 | 30 | 80 | 86 | 381 |
| 6203059 | 3 | 7305670+77 | 30 | 80 | 186 | 825 |
| 6203059 | 4 | 7305673+76 | 30 | 80 | 181 | 802 |
| 6203059 | 5 | 7305674+76 | 30 | 80 | 181 | 802 |

## Lane Occupation

The Data pertaining to the boxes includes information regarding the way the boxes are sorted in the lanes. Two notations are derived from this information and added to the product\_data Table, which are briefly discussed below.

= Box\_sorting. This Represents how the box is sorted. Some boxes are sorted by width and others are sorted by length. If a box is sorted by width, this means the width of the runway is adapted to the width of the box type, when the box is assigned to the runway/lane. If a box is sorted by length, this means the width of the runway/lane is adapted to the length of the box type, when the box is assigned to the runway.

= Box\_not\_sorting. This Represents the inverse of box sorting bi. If a box is sorted by width, then the box\_not\_sorting is the length of the box. This value is used for the rack length calculations, to determine the number of boxes of a type and item, that can fit in one lane. In the same way, if a box is sorted by length, then the inverse is the width which is used in the rack length calculations.

The rack length is then used in combination with the Box\_sorting and the Box\_not\_sorting to determine the following;

Both EOQs added to the table

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| N0 | material ID | ordering\_cost | eoq.min | eoq.max | b\_sorting | b\_not\_sorting |
| 1 | 7305667+74 | 75 | 39 | 180 | 396 | 297 |
| 2 | 7305669+77 | 80 | 86 | 381 | 594 | 396 |
| 3 | 7305670+77 | 80 | 186 | 825 | 594 | 396 |
| 4 | 7305673+76 | 80 | 181 | 802 | 594 | 396 |
| 5 | 7305674+76 | 80 | 181 | 802 | 594 | 396 |

E.g Using box\_ID 6203060, sorted by length , Also for material ID 7305667+74 with therefore,

*1.* Number of boxes in a lane:

*2.* How many lanes part will occupy if you order some number of boxes

$$lane\_i=q\_i \cdot \frac {b\_i^{-1}}{rack\_{length}}=180 \cdot \frac {297}{6000}=8.91= 9 \space lanes $$

number of lanes per item

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| lanes | 9 | 26 | 55 | 53 | 53 | 33 | 52 | 103 |

1. Do we meet the capacity contraint?

*3.* Total number of lanes constraints

Collapsing Rack 4 levels in a rack and joining the 8 racks to become 1 level.

Total rack width available:

Note: rack\_total\_width which is 56,000 mm, looking at the 9 patterns all of which have 150 mm in waste, meaning all 32 levels will have 150 mm waste each. Therefor thereby changing rack total width 51,200.

These *9 patterns* will be explained later in the chapter.

Number of lanes that minimizes cost

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Min lane | 2 | 6 | 13 | 12 | 12 | 8 | 12 | 24 | 4 | 5 | 4 | 14 |
| Max lane | 9 | 26 | 55 | 53 | 53 | 33 | 52 | 103 | 18 | 22 | 22 | 61 |

Min lane: This lanes was derived from .

Max lane: derived from .

* *Overflow*

This is how much we are lacking in capacity assuming we considered optimum lanes thus optimum quatities for all our items when placing an order.

* Overflow in mm

Expressing it in mm terms

- relative overflow in %

In percentage terms

Rack Capacity overflow in mm and percentage

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | mm\_lane\_min | mm\_lane\_max | % lane\_min | % lane.max |
| Overflow | 218694 | 1173092 | 427.1367 | 2291.195 |

*As seen above*, the capacity is exceeded by lots of margin even without adding overall ordering capacity it still exceeds by

* *Introducing capacity constraint*

x

269894 mm is the total expected to be occupied

|  |
| --- |
| x |
| Capacity constraint violated by: 218694 |

As seen above contraint was voilated by , thus we don’t have that kind of capacity for that, therefore we need to optimize to fit our capacity.

Using our example to get number of the total width i.e (summation of lanes) in : Summation of lanes can simply be:

Using equation (1) to prove this concept.

1. Adjust Q by reducing it:

number of orders for part .

average inventory for part

## Constrained EOQ optimization

Subject to:

## ROI: R Optimization Infrastructure

## Registered solver plugins: nlminb, alabama, deoptimr, deoptim, glpk, quadprog, symphony.

## Default solver: auto.

Cost Values and capacities

|  |  |  |
| --- | --- | --- |
|  | Cost Function (€) | Capacity in (mm) |
| Eoq Max | 172279.7 | 1224292 |
| Eoq Min | 124501.5 | 269894 |
| As seen ab | ove, capcities of bo | th EOQ’s exceeds the our rack capacity of 56,000 mm. |

Now adding capacity constraint

## [1] TRUE

Capacitated values for cost and Quantity

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| cost | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 205079.8 | 16 | 20 | 20 | 20 | 20 | 19 | 15 | 15 | 18 | 19 | 18 | 20 | 20 | 19 |

removing waste of 150 for every 32 levels

## No constraint

## Compare results

costs

|  |  |  |
| --- | --- | --- |
|  | Cost Function (€) | capacitaty |
| Eoq Max | 172280 | 1224292 |
| Optimized Q | 202280 | 51132 |

Optimum costs with no capacity constraint using EOQ.max considers is €172279.7, with no capacity constraint optimized using EOQ.min is €124508.6, with capacity constraint, it costs €205079.8 This cost is expected to be higher when capacity constraint is included, as limited capacity of the rack will not allow to take advantage of cost savings.

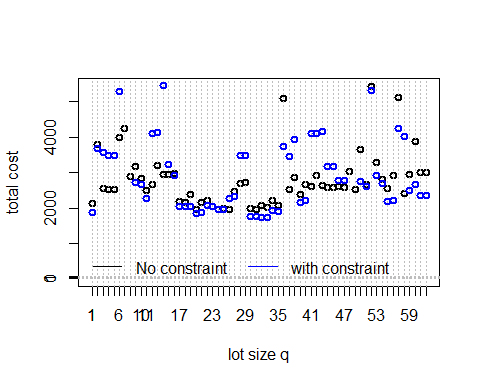
* Plot for lot sizes of costs EOQ Max vs Optimized for capacity

cost function per item

|  |
| --- |
| x |
| It costs € 32800 extra due to lack of capacity |

The above result shows that If there is no capacity constraint, the cost will be lower.

|  |
| --- |
| x |
| 26 Items costs more or same when capacity becomes a problem |



* Notice that 26 blue dots which represents the more expensive items when capacity is accounted for.
* Also the notice , for most items where the blue is larger than black (non constraint items)(the variance is much more) the difference is much larger. the reverse is the case when black is larger than blue.

# Joint Ordering (JO)

Using Joint replenishment problem

Assumptions:

* One supplier with outbound storage.
* products
* Demand rates:
* Stock\_holding cost rates:
* Specific setup costs:
* General setup costs:
* Cycle time of product :

## Objective Function with no capacity constraint

* basic cycle time
* Holding cost multiplier are $H\_0 \space and \space H\_i$
* subject to:

–> an important side note is that the JRP is basically a quite standard EOQ problem which uses the substitution , i.e., . Then, you substitute ans result in the JRP (adding the common ordering cost).

* *Cycle Time*

we determine and

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 52 | 57 | 36 | 7 |
|  | 50.0000000 | 65.0000000 | 65.0000000 | 80.0000000 |
|  | 9386.4320000 | 7724.8080000 | 7652.6450000 | 4374.0900000 |
|  | 0.0729852 | 0.0917303 | 0.0921618 | 0.1352387 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 52 | 57 | 36 | 7 |
|  | 0.0053268 | 0.0084144 | 0.0084938 | 0.0182895 |
|  | 0.1651320 | 0.0943824 | 0.0678407 | 0.0604023 |

$$B=\sqrt \frac {\sum\_{i=0}^n \frac {c\_i^{or}}{m\_i}}{\sum\_{i=0}^n m\_i \cdot H\_i} $$

## [1] 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38  
## [26] 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 5 | 2 | 57 | 36 | 7 | 60 | 6 |
|  | 0.8 | 1.01 | 1.01 | 1.49 | 1.53 | 1.59 |
|  | 1.0 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 |
|  | 1.0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

break occured after 13

Reoptimizing basic cycle time yields and the total cost are 4.99433410^{4}

The order quantities are given by multiplying the cycle times with demand rates , i.e.  such that

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 52 | 57 | 36 | 7 | 60 | 6 |
|  | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
|  | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 |
|  | 133.87 | 71.38 | 54.95 | 118.97 | 23.79 | 89.28 |

–> Now the question is: Is this feasible? –> no:

cost with no capacity constraint

|  |  |  |
| --- | --- | --- |
|  | Cost(€) | Capacity (mm) |
| Not constrained | 49943 | 281834 |

Though there is alot of cost savings, but the capacity required to meet this cost far exceeds the current capacity on ground.

When you try to optimize the JRP directly, without introducing the substitution you need to update the JRP function as follows

cost with no capacity constraint using cycle Time

|  |  |  |
| --- | --- | --- |
|  | Cost(€) | Capacity (mm) |
| Not constrained | 151952 | 1104014 |

Compared to the previous solution, this is far more inefficient.As It costs more and also required more capacity.

## Including capacity constraint

–> You should integrate the shelf capacity constraint while still using basic period approach as it so far has the lowest cost when not constrained.

This is formally defined as

Thus, with the substitution above, the left-hand side changes to

Thus, you can change the constraint function quite easily

Then, you can use the original JRP function and update the model by including the capacity constraint. Therefore I recommend that you fix to some appropriate value based on the feasible solution above

cost with capacity constraint using Basic period approach

|  |  |  |
| --- | --- | --- |
|  | Cost(€) | Capacity (mm) |
| Capacitated | 268254 | 51132 |
| Though more ex | pensive, t | his met the capacity. |

First 7 items in order before reordering

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 52 | 57 | 36 | 7 | 60 | 6 |
|  | 1.00 | 2.00 | 3.00 | 1.00 | 7.00 | 2.00 |
|  | 0.01 | 0.02 | 0.03 | 0.01 | 0.07 | 0.02 |
|  | 14.00 | 15.00 | 18.00 | 13.00 | 18.00 | 19.00 |

First 7 items in order After reordering

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 24.00 | 4.00 | 4.00 | 4.00 | 4.00 | 2.00 |
|  | 0.23 | 0.04 | 0.04 | 0.04 | 0.04 | 0.02 |
|  | 16.00 | 20.00 | 20.00 | 19.00 | 19.00 | 19.00 |

* *Order by m*

After reordering

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | 16 | 20 | 20 | 19 | 19 | 19 | 13 | 13 | 18 |

find a layout scheme such that a precise shelf layout results.

calculate the number of lanes required for each product given a certain order quantity (integer number of boxes):

whereby is the number of boxes per lane dedicated to product . I.e.,

. Thus, for the solution of the JRP we have:

number of lanes for each items

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| lanes | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 2 |

Now, these lanes have to be assigned to the levels of the shelves. Therefore, we first need to determine the types of lanes required. The lanes are just described by their width. Due to the safety margins we should round the lane width to full centimeter. I.e., we need to assign lanes with the following widths, and

Now comes the tricky part: We have to decide how many lanes of a certain width should be assigned for each level. Luckily, there is only a small number of useful patterns of lanes per level. To be precise there are 9 efficient patterns to arrange these 3 lane types (assuming we use each level exhaustively):

Total of 9 patterns

|  |  |  |  |
| --- | --- | --- | --- |
|  | 200 | 400 | 600 |
| 1 | 8 | 0 | 0 |
| 2 | 0 | 4 | 0 |
| 3 | 0 | 1 | 2 |
| 4 | 2 | 0 | 2 |
| 5 | 6 | 1 | 0 |
| 6 | 4 | 2 | 0 |
| 7 | 2 | 3 | 0 |
| 8 | 3 | 1 | 1 |
| 9 | 1 | 2 | 1 |

Let indicate the number of lanes of type associated to pattern . Now we need to assess the number of lanes per required of each type. As outlined above, we know the number of lanes per product , thus we can deduce the demand for lane type () by summing up the for each lane type, i.e., :

number of lanes per required of each type

|  |  |  |  |
| --- | --- | --- | --- |
|  | 200 | 400 | 600 |
| 1 | 14 | 15 | 70 |
| This | misfit | was d | ue to approximations in the on lane assignment by making fraction value to ceiling. |

Need to reduce 70 units to 64 for type 600.

reduce lane assignment while fulfilling demand, to do this, we need to change some values to their floor values. but this can cause backorder, that is inability to meet demand.

lanes in fraction

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| l(qi) | 0.8 | 1.333333 | 1.333333 | 1.266667 | 1.266667 | 0.95 |

To reduce the effect of backorder, as seen above, items 15, 17, 18, 50,57 and 58 are closest decimals to there floor.

selected items

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 15 | 17 | 18 | 50 | 57 | 58 |
| l(qi) | 1.066667 | 1.133333 | 1.133333 | 1.133333 | 2.142857 | 2.142857 |

, , , ,,

Number of lanes for 200, 400 and 600

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| N0 | mat | erial ID b\_ | sorting b\_ | not\_sorting nu | mber\_of\_lanes |
| 15 | 15 | 7305865+77 | 594 | 396 | 1 |
| 17 | 17 | 7305971+75 | 594 | 396 | 1 |
| 18 | 18 | 7305972+75 | 594 | 396 | 1 |
| 50 | 50 | 7343137+72 | 594 | 396 | 1 |
| 57 | 57 | 7455633+73 | 596 | 794 | 2 |
| 58 | 58 | 7455634+73 | 596 | 794 | 2 |

|  |
| --- |
| ld |
| 14 |
| 15 |
| 64 |

* *Changes made for lane assignment*

1. rack\_total\_width which was 56,000 mm, looking at the 9 patterns all of which have 150 mm in waste, meaning all 32 levels will have 150 mm waste each. Therefor thereby changing rack total width 51,200.
2. These changes then affects the values of q for all the 62 items involved.

pat<-data.frame(patterns[3:4,])   
colnames(pat)<-c("200","400","600")  
  
kable(pat,caption = "Patterns needed", row.names = T, escape = F,format = "pandoc")

Patterns needed

|  |  |  |  |
| --- | --- | --- | --- |
|  | 200 | 400 | 600 |
| 1 | 0 | 1 | 2 |
| 2 | 2 | 0 | 2 |

* *Assign Patterns*

Pattern Assigned for each level

|  |  |  |  |
| --- | --- | --- | --- |
|  | 200 | 400 | 600 |
| 1 | 0 | 1 | 2 |
| 2 | 0 | 1 | 2 |
| 3 | 0 | 1 | 2 |
| 4 | 0 | 1 | 2 |
| 5 | 0 | 1 | 2 |
| 6 | 0 | 1 | 2 |
| 7 | 0 | 1 | 2 |
| 8 | 0 | 1 | 2 |
| 9 | 0 | 1 | 2 |
| 10 | 0 | 1 | 2 |
| 11 | 0 | 1 | 2 |
| 12 | 0 | 1 | 2 |
| 13 | 0 | 1 | 2 |
| 14 | 0 | 1 | 2 |
| 15 | 0 | 1 | 2 |
| 16 | 0 | 1 | 2 |
| 17 | 2 | 0 | 2 |
| 18 | 2 | 0 | 2 |
| 19 | 2 | 0 | 2 |
| 20 | 2 | 0 | 2 |
| 21 | 2 | 0 | 2 |
| 22 | 2 | 0 | 2 |
| 23 | 2 | 0 | 2 |
| 24 | 2 | 0 | 2 |
| 25 | 2 | 0 | 2 |
| 26 | 2 | 0 | 2 |
| 27 | 2 | 0 | 2 |
| 28 | 2 | 0 | 2 |
| 29 | 2 | 0 | 2 |
| 30 | 2 | 0 | 2 |
| 31 | 2 | 0 | 2 |
| 32 | 2 | 0 | 2 |
| sum | 32 | 16 | 64 |

* *Assign Items to racks*

## Illustration

|  |  |  |  |
| --- | --- | --- | --- |
| Mi= | 1 Mi= | 2 Mi= | 3 |
| 1 | 7,8,48,49,52 | 6,14,38,43,57,58 | 12,13,15,36,37,41,42,53 |
| 2 | 7,8,48,49,52 | 0 | 0 |
| 3 | 7,8,48,49,52 | 6,14,38,43,57,58 | 0 |
| 4 | 7,8,48,49,52 | 0 | 12,13,15,36,37,41,42,53 |
| 5 | 7,8,48,49,52 | 6,14,38,43,57,58 | 0 |

Items in Mi=1 has 32 order frequency, While Mi=2 has 16 order frequencies in

display Item 4 , having lotsize of 19

item 4

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | eoq.min | eoq.max | b\_sorting | b\_not\_sorting | number\_of\_lanes | Order\_frequency\_M | Lot\_size\_q |
| 4 | 181 | 802 | 594 | 396 | 2 | 4 | 19 |
| \* \*D | emand vs Q | uantity\* |  |  |  |  |  |

which is the capacity of a box for part . Demand in Years for item Demand per day in boxes for item . Demand per day in items for item $yb\_i= $

cycle time is number of days it takes before a new order.

E.g Item 4 has lot size quantity of 19 boxes, and daily demand in boxes is 1.8

item 4 has order frequency M of 4, and is value is 9.9 then floor value is 9

* *Explain formular for demand per day in boxes*

item 4

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | number\_of\_lanes | Order\_frequency\_M | Lot\_size\_q | deman\_per\_day\_boxes | cycle\_time\_in\_days |
| 4 | 2 | 4 | 19 | 1.875 | 9.903517 |

As seen above, cycle time in days in 9.9 days, but we need to round this to the nearest floor

cycle time for each item

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 |
| M | 24.00000 | 4.000000 | 4.000000 | 4.000000 | 4.000000 |
| days | 59.44375 | 9.888421 | 9.888421 | 9.903517 | 9.903517 |

give them floor values

cycle time for each item

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| M | 24 | 4 | 4 | 4 | 4 | 2 | 1 | 1 | 7 | 7 | 7 | 3 |
| days | 59 | 9 | 9 | 9 | 9 | 4 | 2 | 2 | 17 | 17 | 17 | 7 |

* *Convert Cycles to days*

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | * 1 | * 2 | * 3 | * 4 | * 6 | * 7 |
| * Days M. | * 2.47 | * 4.94 | * 7.42 | * 9.89 | * 14.83 | * 17.3 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | * 7 | * 8 | * 10 | * 11 | * 14 | * 15 | * 17 |
| * Days M. | * 17.3 | * 19.78 | * 24.72 | * 27.19 | * 34.61 | * 37.08 | * 42.03 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| * 17 | * 18 | * 21 | * 22 | * 24 | * 31 |  |
| * Days M. | * 42 | * 44 | * 52 | * 54 | * 59 | * 77 |

* *Prove that demand is always met*

Using (T,S) policy

We are using periodic review as order intervals can be derived from order frequency for each of the items.

* Assuming demand is uniformly distributed thus the same quantity of demand repeats each time.
* Initial stock level for the 62 items are given based on lanes assigned
* Each Item has some form of order frequency which we derived cycle time from, going over a time period of 262 days. This represents order interval.
* Lead time is zero.
* At each order interval, we order up to which is the stock level based on lane assigned for each item meaning, lanes are filled.
* Backorder is allowed but customers are willing to wait
* Stock is filled to the capacity in period 1.

*Notations and Formulars*

On-hand stock S(t)

Outstanding orders O(t)

Backorders B(t)

Inventory level available units

In the code below, Shows (T,S) Policy, how demand is met over 262 time period. Reason for TS policy is that we can not exceed capacity, thus we order upto Lane capacity of each item. Also each item has its cycle time in days which was derived from order frequency M.

is the stock level, gotten from lane assigned to each items, the floor of stock level ensures that complete boxes are assigned to a lane. e.g

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| mat | erial ID b | ox ID nu | mber\_of\_lanes b\_ | not\_sorting |
| 4 | 7305673+76 | 6203059 | 2 | 396 |
| using | item 4 as an | example ag | ain |  |

That is 30 boxes in 2 lanes as a lane contains 15 boxes of item 4

Stock level derived from lanes assigned per item

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Stock level | 20 | 30 | 30 | 30 | 30 | 20 | 15 | 15 | 30 | 30 |

Periodic Inventory TS Policy

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| yt | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 |
| qt | 30.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 19.000 | 0.000 |
| lt | 28.125 | 26.250 | 24.375 | 22.500 | 20.625 | 18.750 | 16.875 | 15.000 | 13.125 | 30.000 | 28.125 |

Periodic Inventory TS Policy

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 17 | 18 | 50 |
| Average Inventory | 12.64326 | 7.060305 | 7.060305 | 7.567987 |
| Alpha Service Level | 100.00000 | 90.458015 | 90.458015 | 93.893130 |

As seen from the result, only Items 17 and 18 has an service level of 90.458% and item 57 has service level of 93.9% and the rest of the items has 100% service level.

make there service level be 100% by ordering earlier.

## [1] "box ID" "N0" "material ID"   
## [4] "demand per day" "pieces/box" "price"   
## [7] "demand\_per\_year" "box\_cost" "ordering\_cost"   
## [10] "eoq.min" "eoq.max" "b\_sorting"   
## [13] "b\_not\_sorting" "number\_of\_lanes" "Order\_frequency\_M"   
## [16] "Lot\_size\_q" "deman\_per\_day\_boxes" "cycle\_time\_in\_days"

|  |  |  |  |
| --- | --- | --- | --- |
| de | man\_per\_day\_boxes Or | der\_frequency\_M cy | cle\_time\_in\_days |
| 17 | 0.4000 | 17 | 42 |
| 18 | 0.4000 | 17 | 42 |
| 50 | 0.9375 | 7 | 17 |

Demand occurs in occurs for items not in boxes as order are done in boxes thus we store in boxes, so 0.4 is good this means the boxes are not filled.

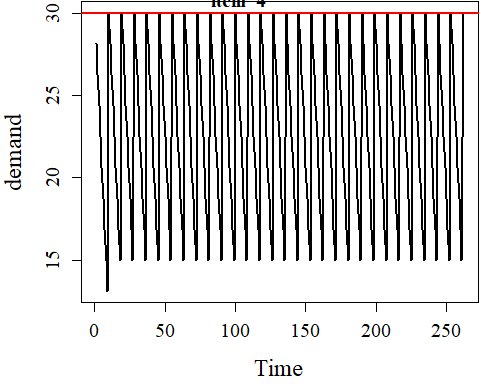
cycle times for items 17 and 18 where assigned 37days, while that of items 50 is 15 days from 7days

Periodic Inventory TS Policy

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 17 | 18 | 50 |
| Average Inventory | 12.64326 | 7.821374 | 7.821374 | 8.484017 |
| Alpha Service Level | 100.00000 | 100.000000 | 100.000000 | 100.000000 |

Periodic Inventory TS Policy

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| yt | 0.9375 | 0.9375 | 0.9375 | 0.9375 | 0.9375 | 0.9375 | 0.9375 |
| qt | 15.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| lt | 14.0625 | 13.1250 | 12.1875 | 11.2500 | 10.3125 | 9.3750 | 8.4375 |

 Using Item 4 as an example, depicted by the red horizontal line, as seen, demand is always met across 262 working days.

## Using S,Q Policy

From Illustration table, Using S,Q policy and Applying JRP. We are lowering the safety stock

If this where to be sq policy, we only order when the quantity is zero.

* We assume that

Check if Lot size q is less than or equal to Stock level

## [1] "item 15 Stock level greater Lot size"  
## [1] "item 17 Stock level greater Lot size"  
## [1] "item 18 Stock level greater Lot size"  
## [1] "item 50 Stock level greater Lot size"

Since 58 items has Lot sizes less than than or equal to lane capacity allocated, this means that in 4 items the lot size quantities is greater than capacity.

This is to ensure that amount ordered is can be contained in the capacity. as

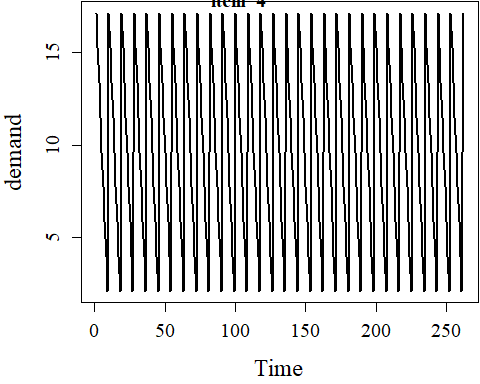
Continous Review Inventory Policy item 4

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 | 1.875 |
| 2 | 19.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 17.125 | 15.250 | 13.375 | 11.500 | 9.625 | 7.750 |

### Service Level

Continous Inventory SQ Policy

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| Average Inventory | 8.436641 | 10.03053 | 10.03053 | 9.653626 |
| Alpha Service Level | 100.000000 | 100.00000 | 100.00000 | 100.000000 |



The table above is Item 1 cycle

Reordering points of this policy is near zero

* Cycle time in days vs Order frequency m

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | * 1 | * 2 | * 3 | * 4 | * 6 | * 7 | * 8 1 | * 0 1 | * 1 1 | * 4 1 | * 5 1 | * 7 1 | * 8 2 | * 1 2 | * 2 2 | * 4 3 | * 1 |
| * Days M. | * 2 | * 5 | * 7 | * 10 | * 15 | * 17 | * 20 | * 25 | * 27 | * 35 | * 37 | * 42 | * 44 | * 52 | * 54 | * 59 | * 77 |

Item 1 has M value of 24 meaning 59 days as cycle time in day, this means therefore, how many cycles in a year(262) meaning total of 4 cycles this can be seen from the diagram its also 4 cycles.

# Analysis

## ABC Analysis

Inventory systems are used to supply subsequent processes with material required to carry out these processes. These processes could be production processes or Customer service, etc. Materials are therefore a necessary evil, held and managed in order to enable other processes. The goal of every organisation therefore is to implement a material provisioning concept, which strives to minimise inventory necessary to provide other services . The characteristics of the materials determines the most appropriate provisioning concepts to be implemented. The problem case in this study uses a storage warehouse provisioning concept. To better understand the characteristics of the materials in this study, we categorise the products by Value, using the ABC analysis. This is because when we talk about stock level and inventory systems related cost, the value of the material is one of the most important drivers. The higher the value, the more costs are implied due to the opportunity cost involved in holding stock.in other words the higher the value the more we should focus on material minimisation As enumerated in the OVGU Inventory management Lecture slides for the ST 2020 by Prof. Dr. T. Kirschstein, the following steps are used to categorize the material by value

Procedure: consumption and price of material , then 1. Calculate value share of material

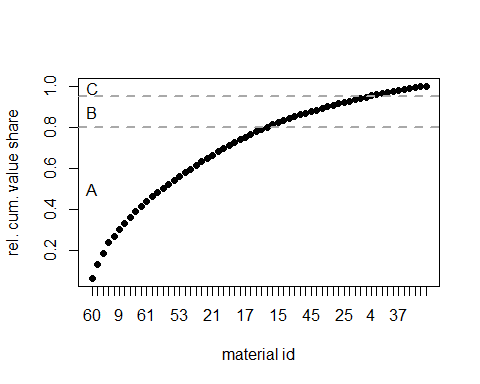
1. Order materials descendingly according to value share:
2. Calculate cumulative ordered value share of each material:
3. Categorize materials according to into classes A,B and C by class limits e.g 80,95,100.

The results of the categorization are summarized in the table below, for the first 3 materials of every class. The full table can be found in the appended R script

ABC analysis results (values classified by boxes)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ord.material.id | price..ord.. | demand..ord.. | mat.values | cum.mat.values | rel.cum.value.shares | class |
| 60 | 256.80 | 1 | 0.07 | 0.07 | 6.7 | A |
| 36 | 252.98 | 2 | 0.07 | 0.13 | 13.2 | A |
| 50 | 215.68 | 1 | 0.06 | 0.19 | 18.8 | A |
| 57 | 196.56 | 3 | 0.05 | 0.24 | 24.0 | A |
| 52 | 127.36 | 6 | 0.03 | 0.27 | 27.3 | A |
| 9 | 121.50 | 1 | 0.03 | 0.30 | 30.4 | A |
| 40 | 116.04 | 0 | 0.03 | 0.33 | 33.4 | A |
| 2 | 114.30 | 2 | 0.03 | 0.36 | 36.4 | A |
| 56 | 110.88 | 1 | 0.03 | 0.39 | 39.3 | A |
| 59 | 95.76 | 1 | 0.02 | 0.42 | 41.8 | A |
| 61 | 87.80 | 1 | 0.02 | 0.44 | 44.0 | A |
| 62 | 87.80 | 1 | 0.02 | 0.46 | 46.3 | A |
| 10 | 76.80 | 1 | 0.02 | 0.48 | 48.3 | A |
| 34 | 76.40 | 0 | 0.02 | 0.50 | 50.3 | A |
| 32 | 75.90 | 0 | 0.02 | 0.52 | 52.3 | A |
| 16 | 75.84 | 1 | 0.02 | 0.54 | 54.2 | A |
| 53 | 73.50 | 2 | 0.02 | 0.56 | 56.2 | A |
| 6 | 72.80 | 4 | 0.02 | 0.58 | 58.0 | A |
| 39 | 68.16 | 0 | 0.02 | 0.60 | 59.8 | A |
| 1 | 67.95 | 0 | 0.02 | 0.62 | 61.6 | A |
| 7 | 66.78 | 5 | 0.02 | 0.63 | 63.3 | A |
| 33 | 63.30 | 0 | 0.02 | 0.65 | 65.0 | A |
| 21 | 63.00 | 0 | 0.02 | 0.67 | 66.6 | A |
| 51 | 62.72 | 1 | 0.02 | 0.68 | 68.2 | A |
| 55 | 60.12 | 1 | 0.02 | 0.70 | 69.8 | A |
| 27 | 59.58 | 1 | 0.02 | 0.71 | 71.3 | A |
| 19 | 55.44 | 1 | 0.01 | 0.73 | 72.8 | A |
| 13 | 51.24 | 2 | 0.01 | 0.74 | 74.1 | A |
| 17 | 49.80 | 0 | 0.01 | 0.75 | 75.4 | A |
| 22 | 47.06 | 0 | 0.01 | 0.77 | 76.6 | A |
| 18 | 46.95 | 0 | 0.01 | 0.78 | 77.8 | A |
| 54 | 46.44 | 2 | 0.01 | 0.79 | 79.1 | A |
| 35 | 45.60 | 0 | 0.01 | 0.80 | 80.2 | B |
| 11 | 45.00 | 1 | 0.01 | 0.81 | 81.4 | B |
| 15 | 44.10 | 2 | 0.01 | 0.83 | 82.6 | B |
| 30 | 37.00 | 0 | 0.01 | 0.84 | 83.5 | B |
| 42 | 35.52 | 2 | 0.01 | 0.84 | 84.4 | B |
| 31 | 32.40 | 0 | 0.01 | 0.85 | 85.3 | B |
| 29 | 32.25 | 2 | 0.01 | 0.86 | 86.1 | B |
| 28 | 31.95 | 2 | 0.01 | 0.87 | 86.9 | B |
| 45 | 30.42 | 2 | 0.01 | 0.88 | 87.7 | B |
| 44 | 30.24 | 2 | 0.01 | 0.89 | 88.5 | B |
| 20 | 30.00 | 0 | 0.01 | 0.89 | 89.3 | B |
| 46 | 29.92 | 2 | 0.01 | 0.90 | 90.1 | B |
| 47 | 28.16 | 2 | 0.01 | 0.91 | 90.8 | B |
| 23 | 27.69 | 0 | 0.01 | 0.92 | 91.5 | B |
| 25 | 27.20 | 0 | 0.01 | 0.92 | 92.2 | B |
| 24 | 24.48 | 0 | 0.01 | 0.93 | 92.9 | B |
| 14 | 24.32 | 4 | 0.01 | 0.94 | 93.5 | B |
| 3 | 24.30 | 2 | 0.01 | 0.94 | 94.1 | B |
| 12 | 24.24 | 2 | 0.01 | 0.95 | 94.8 | B |
| 4 | 24.16 | 2 | 0.01 | 0.95 | 95.4 | C |
| 5 | 24.16 | 2 | 0.01 | 0.96 | 96.0 | C |
| 38 | 22.56 | 4 | 0.01 | 0.97 | 96.6 | C |
| 41 | 21.00 | 2 | 0.01 | 0.97 | 97.1 | C |
| 43 | 20.57 | 3 | 0.01 | 0.98 | 97.7 | C |
| 37 | 20.54 | 2 | 0.01 | 0.98 | 98.2 | C |
| 48 | 17.25 | 6 | 0.00 | 0.99 | 98.7 | C |
| 8 | 17.16 | 5 | 0.00 | 0.99 | 99.1 | C |
| 26 | 13.59 | 1 | 0.00 | 0.99 | 99.5 | C |
| 58 | 12.80 | 3 | 0.00 | 1.00 | 99.8 | C |
| 49 | 7.85 | 6 | 0.00 | 1.00 | 100.0 | C |

A visual representation of the results is also shown in the graph below.



From the graph above, we can see that although there are lots of materials with low values (Class A), the number of high valued materials (Class B and C) is quite significant. Given that these materials are all kept in inventory, there is a need to develop an optimal inventory planning system, such that the costs involved in holding these stocks in inventory will be minimised as much as possible while also ensuring that all demand is fulfilled . The next section of this report gives an overview of such a model.

## Cost Analysis

Total cost Matrix

|  |  |  |
| --- | --- | --- |
|  | unconstrained | constrained |
| Separate Ordering | 174363 | 206266 |
| Joint Ordering | 49943 | 268254 |

in the matrix above, it costs more to When capacity is an issue for both seperate Ordering and Joint Ordering. When non capacited ordering, there is far more cost savings than Separate ordering.

The eoq value for the 62 items are greater than optimized values.

creating a matrix of length 40 filling it with sequence of order quantity values from eoq to optimized order quantity

Items and samples

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Items | 1 | 2 | 3 | 4 | 5 |
| part 1 | 1 | 180 | 175.7949 | 171.5897 | 167.3846 | 163.1795 |
| part 2 | 2 | 381 | 371.7436 | 362.4872 | 353.2308 | 343.9744 |
| part 3 | 3 | 825 | 804.3590 | 783.7179 | 763.0769 | 742.4359 |
| part 4 | 4 | 802 | 781.9487 | 761.8974 | 741.8462 | 721.7949 |
| part 5 | 5 | 802 | 781.9487 | 761.8974 | 741.8462 | 721.7949 |

* *Separate Ordering*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | * sum | * 1 | * 2 | * 3 | * 4 | * 5 | * 6 | * 7 |
| * Non Constrained | * 35235.00 | * 180 | * 381 | * 825 | * 802 | * 802 | * 652 | * 787 |
| * Constrained | * 1140.33 | * 16 | * 20 | * 20 | * 20 | * 20 | * 19 | * 15 |

As seen from the table, with no constraint you tend to order far more than constrained.

* *JRP*

## 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26   
## 8 1 5 5 5 1 1 4 3 4 4 4 3 4 3 3 8 8 4 10 7 7 9 11 10 11   
## 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52   
## 5 4 4 8 9 7 7 7 9 1 4 1 6 4 5 4 4 5 5 4 4 3 5 1 4 1   
## 53 54 55 56 57 58 59 60 61 62   
## 1 3 4 1 1 5 3 1 1 1

in JRP for basic cycle, 47 items has equal or more order frequency when capacitated compared to no capacity constrained.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| su | m 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |  |
| unconst.m | 285 | 8 | 1 | 5 | 5 | 5 | 1 | 1 | 4 | 3 | 4 | 4 | 4 |
| const.m | 544 | 24 | 4 | 4 | 4 | 4 | 2 | 1 | 1 | 7 | 7 | 7 | 3 |
| In JRP , sum | of co | nstra | ined | and u | ncons | train | ed va | lues | and t | here | order | freq | uency values |

for the quantities of all 62 items , unconstrained is greater than constrained

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| sum 1 | 2 |  | 3 | 4 | 5 6 |  |
| 8654.099 | 51 | 48 | 238 | 223 | 223 | 89 |
| 1078.000 | 16 | 20 | 20 | 19 | 19 | 19 |